Nonrenormalization of flux superpotentials in string theory

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ABSTRACT: Recent progress in understanding modulus stabilization in string theory relies on the existence of a non-renormalization theorem for the 4D compactifications of Type IIB supergravity which preserve $N = 1$ supersymmetry. We provide a simple proof of this non-renormalization theorem for a broad class of Type IIB vacua using the known symmetries of these compactifications, thereby putting them on a similar footing as the better-known non-renormalization theorems of heterotic vacua without fluxes. The explicit dependence of the tree-level flux superpotential on the dilaton field makes the proof more subtle than in the absence of fluxes.

Contents

1. Introduction

Four-dimensional theories with $N = 1$ supersymmetry are completely characterized at low energies by three functions — a Kähler potential $K(\varphi, \varphi^*)$, a superpotential $W(\varphi)$ and a gauge-kinetic function $f_{ab}(\varphi)$. The arguments of these functions are the complex scalar fields, φ^i, which appear within the chiral matter multiplets of these theories, and supersymmetry dictates that both W and f_{ab} are holomorphic functions of these arguments.

Although the Kähler function receives corrections order-by-order in perturbation theory in these theories, the superpotential does not and the gauge-kinetic function typically only does at one loop. These properties for W and f_{ab} are known as non-renormalization theorems, and because W controls the vacua of the theory they play a crucial role in understanding these vacua and in particular the circumstances under which supersymmetry can spontaneously break.

These theorems were originally proven using the detailed properties of supersymmetric perturbation theory about flat space [1], but a more robust understanding was achieved with the derivation of the non-renormalization results as a consequence of symmetry arguments combined with the holomorphic dependence on φ^i which supersymmetry requires of W and f_{ab} [2–5]. Among the advantages of the symmetry formulation is the ability easily to extend the results to curved spacetimes and to string theory.

1.1 Non-renormalization for string vacua

For four-dimensional compactifications of heterotic vacua which are $N = 1$ supersymmetric the non-renormalization argument for W is very simple to state [2]. It is based on the observation that for these vacua the two string perturbative expansions — in the string coupling, λ_s, and in low-energies, α' — correspond in the supersymmetric low-energy theory to an expansion in powers of two of the effective theory's scalar fields. These fields are

the 4D dilaton, e^ϕ, and the field σ which describes the volume of the underlying Calabi-Yau compactification. For heterotic vacua these fields combine with two low-energy axions into two complex fields, S and T, which are the scalar components of two matter supermultiplets, and at lowest order a direct dimensional reduction shows that neither of these fields appears in the low energy superpotential [6, 3].

The non-renormalization argument then proceeds as follows. At higher orders the dependence of W and f_{ab} on S and T is dictated by holomorphy and low-energy symmetries [3, 2]. In particular, there is a low-energy Peccei-Quinn (PQ) symmetry $S \to S + i\omega$ (where ω is a constant parameter), which forbids W from depending on $\operatorname{Im} S$ to all orders in perturbation theory. However since W must be a holomorphic function of S, this also precludes W from depending on $\operatorname{Re} S \propto e^\phi$, which is the string coupling constant. We conclude from this that the PQ symmetry precludes W from developing a dependence on S, and so ensures that W receives no corrections in string perturbation theory.

The α' expansion similarly involves the field σ contained within $\operatorname{Re} T$, and a similar argument involving another axion-like symmetry which shifts $\operatorname{Im} T$ also precludes W from acquiring a dependence on T. The same symmetry allows f_{ab} to receive T-dependent contributions, but these must be independent of S and so arise only at one string loop. It precludes corrections to f_{ab} from arising at higher loops. Arguments such as these show how symmetries can imply the non-renormalization of W and allow only a limited renormalization of f_{ab}. Their implications are typically restricted to perturbation theory because nonperturbative effects can break the underlying symmetries. Even in this case consistency with the known anomalies in the underlying axionic symmetries requires that the dependence acquired by W must be purely exponential in S and T.

In recent years Type IIB vacua have become the focus of much attention, due to their utility in addressing the long-standing problem of the stabilization of string moduli [7]. $N = 1$ supersymmetric 4D compactifications of Type IIB vacua are useful for modulus stabilization because in the presence of background fluxes they generate a superpotential, W, which can stabilize all of the complex-structure moduli of the underlying warped Calabi-Yau geometry [10] (see also [11]). For compactifications with constant dilaton this superpotential has a specific (Gukov-Vafa-Witten) form [8], W_{GVW}, which allows a simple expression (see eq. (2.11)) in terms of geometrical quantities defined on the Calabi-Yau internal space. For F-theory compactifications for which the dilaton varies across the internal dimensions the superpotential is more complicated, but is also known [8, 12].

What is important about the Type-IIB moduli-stabilization arguments of ref. [7] is that they are done in a controllable way, within the domain of validity of the λ_s and α' expansions. The non-renormalization of W plays an important part in doing so, since it is what precludes the appearance of corrections to W order-by-order in λ_s and α'. Unfortunately, a justification of the non-renormalization theorem for Type-IIB vacua along the lines used for heterotic vacua has not yet been made, largely because symmetries cannot preclude W_{GVW} from depending on the string coupling due to the explicit presence on the dilaton, e^ϕ, which is already present in the lowest-order superpotential, W_{GVW}. We note for later purposes that although this dilaton dependence complicates the non-renormalization

argument for the string-coupling expansion, we *do* know that W_{GVW} does not receive any corrections within the α' expansion just as for heterotic vacua, because it cannot depend explicitly on the Kähler-structure moduli.

Our purpose in this note is to fill in this step by providing, in section 2 below, a simple derivation of the non-renormalization theorem for Type IIB vacua based only on holomorphy and symmetries. Besides filling in a missing step in the modulus-stabilization arguments this renormalization theorem can also have other applications, some of which we briefly describe in section 3.

2. Non-renormalization for type IIB vacua

In this section we explain how the non-renormalization theorem follows from holomorphy and other incidental symmetries of the low-energy effective 4D action. We do so by starting with a brief summary of the low-energy field content and a description of the low-energy symmetries on which our arguments are based.

2.1 10D type IIB supergravity

The massless sector of Type IIB string compactifications is described by ten-dimensional Type IIB supergravity, which has two supersymmetries generated by two Majorana-Weyl supercharges which share the same 10D chirality. This supersymmetry algebra also enjoys a U(1) R-symmetry which rotates these two supercharges into one another. The field content of the theory consists of the graviton, g_{MN}, with R-charge $q = 0$; a complex Weyl gravitino, ψ_M, with $q = 1/2$; the NS-NS and R-R 2-forms, B^1_{MN} and B^2_{MN}, with $q = 1$; a complex Weyl dilatino, λ, with $q = 3/2$; two scalars (the dilaton, e^ϕ, and the R-R 0-form, $C_{(0)}$), with $q = 2$; and the self-dual 4-form $C_{(4)}$, with $q = 0$.

The bosonic part of the 10D low-energy effective theory for the Type IIB sector has the following form in the string frame [10], to lowest order in α' and string coupling:

$$S_{\rm S} = \frac{1}{2} \int d^{10}x \, \sqrt{-g_s} \left\{ e^{-2\phi} \left[R_s + 4(\partial\phi)^2 \right] - \frac{1}{2} F^2_{(1)} - \frac{1}{12} G_{(3)} \overline{G}_{(3)} - \frac{1}{480} F^2_{(5)} \right\} \qquad (2.1)$$

$$- \frac{i}{8} \int e^\phi \, C_{(4)} \wedge G_{(3)} \wedge \overline{G}_{(3)} + \sum_b \left[-\mu_b \int_b d^{(p+1)}y \, e^{-\phi} \sqrt{-\hat{g}_s} + \mu_b \int_b \hat{\mathcal{C}}_{(p+1)} \right] ,$$

where $F_{(p+1)} = {\rm d}C_{(p)} \ldots$, $\tau = C_0 + i e^{-\phi}$, $H^\alpha_{(3)} = {\rm d}B^\alpha_{(2)}$ and $G_{(3)} = \tau H^1_{(3)} + H^2_{(3)}$. The sum on 'b' runs over all of the various Dp-branes which source the bulk fields, and $\hat{g}$ and $\hat{\mathcal{C}}$ denote the pull-back to the brane world-sheet of the bulk metric and the appropriate $(p + 1)$-form which defines the brane Wess-Zumino interaction with the RR potentials.

It is useful to reformulate the above action (2.1) by defining the Einstein metric $g_{MN} = e^{-\phi/2}g_{sMN}$, and therefore the action becomes

$$S_{\rm E} = \frac{1}{2} \int d^{10}x \, \sqrt{-g} \left\{ R - \frac{\partial_M \tau \partial^M \tau}{2(\operatorname{Im}\tau)^2} - \frac{G_{(3)} \cdot \bar{G}_{(3)}}{12 \operatorname{Im}\tau} - \frac{1}{480} F^2_{(5)} \right\} \qquad (2.2)$$

$$- \frac{i}{8} \int \frac{C_{(4)} \wedge G_{(3)} \wedge \bar{G}_{(3)}}{\operatorname{Im}\tau} + \sum_b \left[-\mu_b \int_b d^{(p+1)}y \, e^{(p-3)\phi/4} \sqrt{-\hat{g}} + \mu_b \int_b \hat{\mathcal{C}}_{(p+1)} \right] .$$

At the classical level the bulk part of this theory enjoys an accidental global $\mathrm{SL}(2,\mathbb{R})$ invariance which plays an important role in our arguments. This symmetry is nonlinearly realized, with the scalars ϕ and C_0 transforming as a coset $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ [13 – 19]. In terms of the complex combination, τ, defined above, the action of this symmetry takes the standard form

$$\tau \to \frac{a\tau + b}{c\tau + d}, \qquad B_{MN}^\alpha \to (\Lambda^T)^{-1\alpha}{}_\beta B_{MN}^\beta \qquad \Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{R}),$$
$$g_{MN} \to g_{MN}, \qquad C_{MNPQ} \to C_{MNPQ}, \tag{2.3}$$

while the fermions transform as

$$\psi_M \to \left(\frac{c\overline{\tau} + d}{c\tau + d}\right)^{1/4} \psi_M \qquad \lambda \to \left(\frac{c\overline{\tau} + d}{c\tau + d}\right)^{3/4} \lambda. \tag{2.4}$$

As usual the real parameters a through d satisfy $ad - bc = 1$.

The field strengths for the bosonic fields can be combined into combinations which also transform simply under the global $\mathrm{SL}(2,\mathbb{R})$ transformations, as follows. In terms of $H_{(3)}^\alpha = \mathrm{d}B_{(2)}^\alpha$ and $F_{(5)} = \mathrm{d}C_{(4)}\ldots$, the combinations

$$P_M = i\,\frac{\partial_M \tau}{2\,\tau_2}, \quad Q_M = -\frac{\partial_M \tau_1}{2\,\tau_2} \qquad \text{and} \qquad G_{(3)} = \tau\, H_{(3)}^1 + H_{(3)}^2 \tag{2.5}$$

inherit the following transformation properties under $\mathrm{SL}(2,\mathbb{R})$:

$$P_M \to \left(\frac{c\overline{\tau} + d}{c\tau + d}\right) P_M, \quad Q_M \to Q_M - \frac{i}{2}\,\partial_M \ln\left(\frac{c\overline{\tau} + d}{c\tau + d}\right), \quad G_{(3)} \to \frac{G_{(3)}}{c\tau + d} \tag{2.6}$$

and $F_{(5)} \to F_{(5)}$.

The R symmetry can be regarded as a subgroup of $\mathrm{SL}(2,\mathbb{R})$, as may be seen by choosing the following one-parameter family of $\mathrm{SL}(2,\mathbb{R})$ transformations:

$$b = -|\tau|, \quad c = 1/|\tau| \quad \text{and} \quad d = 0, \tag{2.7}$$

for any given τ. These choices satisfy $ad - bc = 1$ and have the property that $c\tau + d = \tau/|\tau| = e^{-i\alpha}$, where $\alpha = -\arg\tau$. With this choice $c\overline{\tau} + d = e^{+i\alpha}$ and so the $\mathrm{SL}(2,\mathbb{R})$ transformation properties of the field strengths and fermions imply these transform under this one-parameter subgroup as

$$P_M \to e^{+2i\alpha}\, P_M, \quad Q_M \to Q_M + \partial_M \alpha, \quad G_{(3)} \to e^{+i\alpha}\, G_{(3)},$$
$$\psi_M \to e^{+i\alpha/2}\, \psi_M \quad \text{and} \quad \lambda \to e^{+3i\alpha/2}\, \lambda, \tag{2.8}$$

which may be recognized as the R transformation properties for each of these fields.

The normalization of the R charge may be obtained by seeing how the supersymmetry parameter, ϵ, transforms, which may be inferred from the above expressions together with the supersymmetry transformation rules [13 – 18]:

$$\delta\lambda = \frac{i}{\kappa}\Gamma^M P_M \epsilon^* - \frac{1}{24\sqrt{\tau_2}}\Gamma^{MNP} G_{MNP}\, \epsilon \tag{2.9}$$

$$\delta\psi_M = \frac{1}{\kappa}D_M \epsilon + \frac{i}{480}\Gamma^{M_1 \ldots M_5} F_{M_1 \ldots M_5}\Gamma_M\, \epsilon - \frac{i}{96\sqrt{\tau_2}}\left(\Gamma_M^{PQR} G_{PQR} - 9\Gamma^{PQ} G_{MPQ}\right)\epsilon^*,$$

where

$$D_M \epsilon = \left(\partial_M + \frac{1}{4}\omega_M^{AB}\Gamma_{AB} - \frac{i}{2}Q_M \right)\epsilon . \tag{2.10}$$

We read off from these expressions that ϵ has R charge $q = \frac{1}{2}$.

2.2 Symmetry breaking

Since our interest is in using these symmetries to constrain the properties of the quantum-corrected superpotential and gauge potential, it behooves us to understand which symmetries survive perturbative quantum corrections. In particular, it is known that the $\mathrm{SL}(2,\mathbb{R})$ symmetry is only a symmetry to leading order in λ_s and α', which does not survive intact into the quantum theory. For this reason we identify several important subgroups of $\mathrm{SL}(2,\mathbb{R})$ which do survive quantum corrections. The three important such symmetries which we use in the following are:

1. *R-invariance:* we have seen that the $\mathrm{U}(1)$ R symmetry of the 10D supersymmetry transformations is a subgroup of $\mathrm{SL}(2,\mathbb{R})$, and is a symmetry at leading order in α' because it is a symmetry of the 10D supergravity given by eq. (2.1). This is not an exact symmetry of the string theory, however, and there is evidence that this symmetry is broken at subleading but finite order in the α' expansion [9]. However if we restrict ourselves to leading order in α', we now argue that this symmetry remains unbroken to all orders in the string coupling, e^ϕ. This follows because 10D supersymmetry completely dictates the dilaton dependence of the two-derivative action, eq. (2.1), which gives the action to leading order in α', showing that the two-derivative terms of the 10D action are not renormalized in string perturbation theory.

2. *Peccei-Quinn (PQ) invariance:* another subgroup of $\mathrm{SL}(2,\mathbb{R})$ which is anomalous but which also survives to all orders in string perturbation theory is the PQ symmetry defined by the $\mathrm{SL}(2,\mathbb{R})$ transformations $a = d = 1$ and $c = 0$, for which $\tau \to \tau + b$.

3. $\mathrm{SL}(2,\mathbb{Z})$ *invariance:* although quantum corrections generically break the $\mathrm{SL}(2,\mathbb{R})$ invariance, they are expected to preserve the discrete subgroup which is obtained if the parameters a, b, c and d are restricted to be integers. This symmetry is expected to survive but only after non-perturbative corrections are included. It will not play an important role in our arguments which are only perturbative.

All three of these symmetries may be used to constrain the form of the low-energy effective action to all orders in perturbation theory. We now turn to a description of their implications for compactifications whose low-energy action is described by an $N = 1$ 4D supergravity.

2.3 The effective 4D supergravity

It is always possible to use the standard supergravity action to describe the dynamics of the low-energy degrees of freedom in any compactification to four dimensions which preserve $N = 1$ supersymmetry in 4 dimensions. The same is true for compactifications which break

4D $N = 1$ supersymmetry, provided the 4D particle supermultiplets of interest are split by less than the Kaluza-Klein (KK) scale.[1]

There is a broad class of 4D compactifications of Type IIB supergravity having an approximate $N = 1$ supersymmetry which are described by a low-energy 4D supergravity [10]. The low-energy fields for these compactifications consist of 4D supergravity coupled to a number of $N = 1$ gauge and matter supermultiplets. The scalar fields of the matter multiplets include the various compactification moduli, various low-energy axion fields, and scalars of other types. Among these massless fields is the volume modulus, σ, whose scalar component contains the extra-dimensional volume field, V, as well as the scalar which is related to V by supersymmetry, obtained in four dimensions by dualizing one of the components, $C_{\mu\nu pq}$, of the 4-form field, $C_{(4)}$ which have two indices in the uncompactified directions, x^μ, $\mu = 0, \ldots, 3$.

As mentioned in the introduction, any 4D $N = 1$ supergravity is characterized by its Kähler function, K, superpotential, W, and gauge kinetic term, f_{ab}. For non-renormalization theorems our interest in particular is in the holomorphic functions W and f_{ab}. For Type IIB theories for which the dilaton field τ varies trivially over the extra dimensions the expression for the superpotential to leading order in string and α' perturbation theory is given in terms of the 10D fields by the Gukov-Vafa-Witten expression [8]:[2]

$$W_{GVW} = \int_M G_{(3)} \wedge \Omega, \tag{2.11}$$

where G_3 is as above and Ω is the unique $(3, 0)$ form of the Calabi-Yau space which underlies the internal six-dimensional space, M_6. This defines a function of the 4D fields which appear in the low-energy theory because Ω depends implicitly on those 4D fields which correspond to the complex-structure moduli of M_6 (but not on all such fields, such as those associated with the Kähler moduli). Similarly, W_{GVW} also depends on τ through the definition, eq. (2.5), of $G_{(3)}$. It is the dependence on these fields of the resulting scalar potential, V_{GVW}, which stabilizes the moduli — including in particular τ — described by these fields.

It is instructive to exhibit explicitly the $\mathrm{SL}(2, \mathbb{R})$ transformation of this leading-order superpotential, which transforms in the same way as does $G_{(3)}$:

$$W_{GVW} \to \frac{W_{GVW}}{c\tau + d}. \tag{2.12}$$

This is as required to make the low-energy action invariant since the variation of W cancels the variation of the Kähler function, K, which has the form

$$K = -\ln(\tau - \overline{\tau}) + \hat{K}, \tag{2.13}$$

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[1]In general, an effective theory can be described by the standard 4D supergravity provided that the low-energy field content can linearly realize the approximate $N = 1$ supersymmetry. This is generically possible if the mass splittings amongst the low-energy multiplets is smaller than the masses of the heavy particles whose removal generates the effective theory in question.

[2]A generalization of this expression applies to F-theory compactifications for which τ varies across the extra dimensions.

where $\hat{K}$ is an $SL(2,\mathbb{R})$ invariant function of the other fields. Since the low-energy action only depends on K and W through the combination $K + \ln|W|^2$, the transformation of the first term in K precisely cancels the transformation of W, given that

$$\tau - \overline{\tau} \to \frac{\tau - \overline{\tau}}{|c\tau + d|^2}\,. \tag{2.14}$$

At this point we can see why the arguments for the non-renormalization of W for heterotic vacua do not straightforwardly apply to Type IIB vacua. Although there is a shift symmetry, $\tau \to \tau + b$, this cannot preclude W from depending on τ because W_{GVW} already has a τ dependence through the τ's which enter into the definition of $G_{(3)}$. This is possible because of the presence of the background fluxes, $\langle H^1_{mnp} \rangle$ and $\langle H^2_{mnp} \rangle$, which also transform under the shift symmetry.[3]

2.4 The non-renormalization theorem

We now show how to use the symmetries given above to derive a non-renormalization result for the Type IIB case. In order to do so we assume that the low-energy 4D effective theory linearly realizes $N = 1$ supersymmetry, and so may be written in terms of the standard $N = 1$ supergravity action, characterized by the functions K, W and f_{ab}.

To start we first require a statement of the field content of the effective 4D theory, and how these fields transform under the global symmetries of the 10D theory. For these purposes we keep track of two kinds of fields. The first kind consists of the fields which describe the light degrees of freedom whose masses are smaller than the KK scale, and so whose dynamics is described by the low-energy $N = 1$ supersymmetric 4D theory. The scalar fields of this theory — which we denote collectively as φ^i — transform under supersymmetry as chiral matter. We also imagine there to be a low-energy gauge group, with gauge multiplets denoted by $\mathcal{W}^a$, as well as the 4D supergravity multiplet itself.

Included among the matter multiplets are both the Kähler-structure moduli (including the volume modulus, σ) which are left massless by W_{GVW}, and the complex-structure moduli (including the dilaton multiplet τ) which appear explicitly in W_{GVW}. The complex-structure moduli can appear in the low-energy effective theory because the masses they acquire because of their presence in the GVW potential are systematically light (for large extra-dimensional volume) compared with the KK scale. These fields can be defined without loss of generality to be invariant under the PQ symmetry, by absorbing into their definition the appropriate power of $e^{i\tau}$.

The second class of fields whose dependence we follow in the low-energy action are 'spurions' [20], which describe the transformation properties of the background flux vacuum-expectation-values (*v.e.v.*'s) under the symmetry transformations of interest. These fluxes reside within the background value for the 2-form, $G_{(3)}$, and may be regarded as the *v.e.v.*s,

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[3]Notice that for topological (and therefore non-perturbative) reasons, the fluxes are quantised breaking the $SL(2,\mathbb{R})$ symmetry to $SL(2,\mathbb{Z})$. Since our arguments are only perturbative we will not make use of this constraint in our coming discussion. We thank S. de Alwis and J. Conlon for helpful discussions on this point.

$\mathcal{G}^r$, of the large collection of 4D scalar fields which are obtained when $G_{(3)}$ is dimensionally reduced. As we have seen, these fields transform nontrivially under the R and PQ symmetries, with $\mathcal{G}^r$ transforming as in eq. (2.6).

Both types of fields are important for the non-renormalization theorem, because string perturbation theory is related to the dependence of the low-energy action on e^ϕ (and so on τ), while the α' expansion is related to the dependence on the volume modulus σ. It is also useful to follow the dependence on the spurions $\mathcal{G}^r$, since these contain τ and transform under the R symmetry.

To establish the non-renormalization theorem, we now argue in two steps. First we restrict our attention to lowest-order in the α' expansion, and argue that the superpotential is not renormalized to all orders in string perturbation theory. For this part of the argument the R symmetry can be regarded to be unbroken and so can be used to constrain the possible form for W. We then separately argue that this result also remains true to all orders in the α'.

To leading order in α', but to all orders in string loops, we may use the R-invariance of W to restrict its form. We may do so because although the R symmetry is broken by string loops, we have argued above that this breaking cannot arise to leading order in α', due to the restrictive form of those terms in the 10D supergravity action involving two or fewer derivatives. One might worry that the R symmetry might be spontaneously broken by the background, if background fields (like fluxes) break the R symmetry. But this is the point of keeping the spurion fields, $\mathcal{G}^r$, which capture how the background fields transform. Provided the fields $\mathcal{G}^r$ are the only backgrounds which break the R symmetry, keeping track of their appearance in the low-energy 4D theory guarantees this low-energy action is R-invariant (to lowest order in α'). The only bosonic background field which can break R invariance and yet is *not* encoded in the $\mathcal{G}^r$'s would be a background value for P_m, and so our analysis does not cover fields for which the dilaton field τ varies across the internal Calabi-Yau space. By tracking only the dependence on $\mathcal{G}^r$ we assume $P_m = 0$, and so restrict our analysis to those orientifold limits of F-theory for which the Gukov-Vafa-Witten flux superpotential, W_{GVW}, applies.

Because the supersymmetry transformation parameter has R-charge $q_\epsilon = +\frac{1}{2}$, it follows that R-invariance of the action implies the superpotential must carry R-charge $q_W = +1$. Since $G_{(3)}$ also carries $q_G = +1$ we are always free to take W to be proportional to one of the $\mathcal{G}^r$s — say $\mathcal{G}^0$ — and so write

$$W(\varphi^i, \tau; \mathcal{G}^r) = \mathcal{G}^0\, w\left(\varphi^i; \frac{\mathcal{G}^r}{\mathcal{G}^0}\right), \tag{2.15}$$

for some function w. w cannot depend separately on τ (beyond the τ dependence of the $\mathcal{G}^r$'s) because w must be PQ-invariant and τ shifts under this symmetry while all of the other arguments of w do not transform. Explicit calculation shows that the lowest-order GVW result, W_{GVW}, corresponds to w being given by a strictly linear function of the arguments $\mathcal{G}^r/\mathcal{G}^0$,

$$W_{GVW}(\varphi^i, \tau; \mathcal{G}^r) = \sum_{r \geq 0} \mathcal{G}^r\, w_r(\varphi^i). \tag{2.16}$$

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To establish the non-renormalization theorem for the string-coupling expansion, we now argue that quantum corrections cannot change the form of eq. (2.16), to all orders in perturbation theory. What makes this argument tricky is the observation that whatever form it takes, it *cannot* be based on arguing W is independent of τ, due to the τ-dependence which already appears through the variables $\mathcal{G}^r$. This τ dependence arises because of the relative factor of τ which distinguishes the NS-NS and RR fields, $H_{(3)}^1$ and $H_{(3)}^2$, inside $G_{(3)}$. In its turn, the τ-dependence of $G_{(3)}$ can be traced to the statement that the string coupling constant, e^ϕ, is *not* the loop-counting parameter for the low-energy 10D supergravity lagrangian governing fluctuations about Type IIB vacua, even in the string frame.[4] This may be seen from the different factors of e^ϕ which arise in the lagrangian of eq. (2.1), as opposed to the corresponding action for heterotic vacua where the dilaton appears as an overall factor of $e^{-2\phi}$ in the string frame.

It is therefore convenient to organize the perturbative series slightly differently, by re-scaling the string-frame fields as follows:

$$e^\phi \to \lambda\, e^\phi, \qquad C_{(p)} \to \lambda^{-1}\, C_{(p)} \tag{2.17}$$

and $y^M \to \lambda^{-1/(p+1)}\, y^M$ (for the brane position, y^M), for constant λ. Under this re-scaling we have

$$F_{(p)} \to \lambda^{-1}\, F_{(p)} \qquad G_{(3)} \to \lambda^{-1}\, G_{(3)} \qquad \sqrt{-\hat{g}}\, d^{(p+1)}y \to \lambda^{-1}\, \sqrt{-\hat{g}}\, d^{(p+1)}y \tag{2.18}$$

and so the action of eq. (2.1) satisfies $S \to S/\lambda^2$. After performing this re-scaling we formally expand all observables (and the low-energy 4D effective action) as a series in λ, setting $\lambda = 1$ at the end of the calculation. Since the action carries an overall factor of λ^{-2} the expansion in powers of λ is simply the loop expansion for the action of eq. (2.1).

Now comes the main point. Since $\lambda \to 1$ at the end of the calculation, we do not claim that it is a good approximation to work to any fixed order in powers of λ. In this regard this makes the series in λ unlike the *string* loop expansion, for which successive terms are suppressed by powers of the small quantity e^ϕ. The λ series simply represents a reorganization of the string-loop expansion in powers of e^ϕ, in which terms are grouped according to their power of λ rather than their power of e^ϕ. However, if we can establish that W is not corrected to *all* orders in λ, then it also follows that it is not corrected to all orders in e^ϕ.

And it is simple to see that W is not corrected from the lowest-order result, eq. (2.16), to any order in λ. This is because under the above rescaling $\mathcal{G}^r \to \lambda^{-1}\mathcal{G}^r$ and so the ratio $\mathcal{G}^0/\mathcal{G}^r$ is λ-independent. It therefore follows from eq. (2.15) that the R and PQ symmetries imply that w is λ independent, and so W transforms in precisely the same way as does W_{GVW}: $W \to \lambda^{-1}\, W$. It follows that W is completely given by its lowest-order approximation, $W = W_{GVW}$, at lowest order in α' but to all orders in the string coupling.

It remains to extend this result to all orders in α', and this part of the argument follows much as for the heterotic string. In particular we know that W cannot depend on any Kähler moduli to any order in α', because this is precluded by a combination of

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[4]We thank J. Polchinski for emphasizing this point.

holomorphy and shift symmetries for the form fields which appear in these moduli. However the dimensionless parameters which control the α' expansion are powers of α' divided by the volume of various cycles of the background Calabi-Yau space, and these volumes are all counted among the Calabi-Yau's Kähler moduli. Since W is independent of these Kähler moduli, it follows that it is also uncorrected to all orders in α'. This establishes the desired result to all orders in both the α' and string-coupling expansions.

At this point the hairs on the back of the reader's neck may be bristling due to a visceral discomfort with re-ordering the loop expansion, which is itself generically divergent.[5] We therefore make a brief parenthetic aside at this point to help put the reader's mind at rest. For these purposes recall that the loop expansion (in either λ or e^{ϕ}) is asymptotic, and so the vanishing of a quantity (like $W - W_{GVW}$) to all orders in the coupling is equivalent to the statement that the quantity vanishes faster than any power of the relevant coupling as the coupling goes to zero. Our goal is to sketch how this may be done for the e^{ϕ} expansion given that it is true for the λ expansion.

The main point is that although the classical action, eq. (2.1), involves more than one overall power of e^{ϕ}, its dependence on e^{ϕ} is not arbitrarily complicated. This is because the entire action corresponds either to tree- or one-loop level in the string expansion (for the NS-NS and RR terms respectively). Consequently, all of the contributions which arise at any finite order in the e^{ϕ} expansion are contained within a higher, but finite, number of terms in the λ series. If it is known that the contributions to any particular quantity vanish to all orders of the λ expansion, it is therefore possible to set up an inductive argument which proves that the same is true to all orders in the expansion in e^{ϕ}.

3. Discussion

We provide in this note a derivation of the non-renormalization theorem for those Type IIB vacua described by the Gukov-Vafa-Witten superpotential, which is similar in spirit to the well-known results for heterotic vacua in that it relies purely on simple symmetry arguments (rather than on more detailed properties of the calculus of perturbation theory in super-space). We regard this to fill in an important missing step in the recent arguments for the existence of discrete de Sitter type vacua for Type IIB string theory.

We have made crucial use not only of the Peccei-Quinn symmetry used in the original Dine-Seiberg proof for the heterotic case, but also of the global R-symmetry as used in Seiberg's proof for the nonrenormalisation of W in $N = 1$ supersymmetric field theories [4, 5].

Notice however that even though in 4D these global symmetries survive at all orders in perturbation theory, therefore guaranteeing the valididty of the theorem to all orders, the argument is a bit more complicated in string theory. The complication arises because global symmetries of the effective, leading-order, 10D action are known to be broken by combinations of α' and string loop corrections and by the compactification process. For instance, such R-symmetry breaking terms are induced within topological string theory,

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[5]We thank Liam McAllister for helping us to drive a silver spike through the heart of this particular demon.

including terms like $\int F^{2g-2} R^2 d^4x$ where F is the self-dual part of the field strength of a graviphoton field in $N = 2$ supergravity and g is the genus of the worldsheet [9]. Similar terms also arise for $N = 1$ vacua.[6]

These symmetry-breaking terms can be seen to be F-terms which appear as higher derivative corrections to the effective action and are therefore not captured by a superpotential. For the validity of our proof, the only requirement that must be satisfied is for these global symmetries to hold to leading order in the α' expansion. The reason being that if the superpotential is not renormalized at leading order in α' then the fact that W is independent of the Kähler moduli (which controls the α' expansion) guarantees that W will not be renormalized in perturbation theory.

The main restriction of our analysis is the assumption of trivial dilaton configurations, with $P_m \propto \partial_m \tau = 0$. This assumption prevents us from directly extending our conclusions to the generalizations of W_{GVW} which govern the low-energy limit of F-theory compactifications [12]. Our argument does not directly apply in this case because the 4D spurions, $\mathcal{P}^r$, describing nonzero P_m carry two units of R-charge, and so allow W to depend on the R-invariant ratios $\mathcal{P}^r/(\mathcal{G}^0)^2$. But this ratio scales like λ^2 and so at face value does not forbid W from acquiring nontrivial changes at finite loop orders.

Although we focus here on the superpotential, we notice in passing that we expect similar arguments to apply to the holomorphic gauge coupling function, f_{ab}. The simplest case to consider is that for which the gauge fields live on a D7 brane, in which case the lowest-order calculation gives a result independent of τ:

$$f_{ab} = \sigma \, \delta_{ab} \,, \tag{3.1}$$

where σ is the appropriate volume modulus for the cycle on which the D7 brane wraps. Since the PQ transformation does not have an anomaly involving these gauge fields we know that f_{ab} must be invariant under PQ transformations. Similarly, R-invariance of the action requires that f_{ab} should also be invariant under the R symmetry, and this implies

$$f_{ab}(\varphi^i, \tau; \mathcal{G}^r) = \mathcal{F}_{ab}\left(\varphi^i; \frac{\mathcal{G}^r}{\mathcal{G}^0}\right), \tag{3.2}$$

where $\mathcal{F}_{ab}$ is the most general R-invariant combination built from φ^i and $\mathcal{G}^r$. From here on the argument proceeds as before, by re-scaling fields by powers of λ so that $S \to S/\lambda^2$ and performing an expansion in powers of λ. It follows that the function $\mathcal{F}_{ab}$ receives no corrections in the string loop expansion compared to the lowest-order result. Notice that this does *not* preclude $\mathcal{F}_{ab}$ from differing from the lowest-order result, eq. (3.1), so long as the difference does not depend on τ. This agrees with the known direct calculations [21], which find that the gauge kinetic function receives nontrivial τ-independent corrections. It would be interesting to consider the case of gauge couplings on D3 branes.

The main part of our argument involves the re-organization of the string loop expansion in terms of the loop expansion of the leading low-energy field theory. It is easier to show that no corrections arise in this second expansion, and we argue that this implies also the

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[6]We thank Nathan Berkovits for calling these articles to our attention.

absence of corrections in the string loops. It is clear that this kind of argument generalizes to other string vacua besides the Type IIB case, for which both NS and Ramond states also arise in the bosonic part of the low-energy field theory. In particular, it would be of great interest to see whether similar considerations are applicable to corrections to the lowest-order F-theory superpotential, $W = \int G_4 \wedge \Omega$, as well as flux superpotentials for heterotic and type IIA strings.

Acknowledgments

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